Extensions of Euler Type II Transformation and Saalschutz's Theorem

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Abstract
In this research paper, motivated by the extension of the Euler type I transformation obtained very recently by Rathie and Paris, the authors aim at presenting the extensions of Euler type II transformation. In addition to this, a natural extension of the classical Saalschutz's summation theorem for the series 3F2 has been investigated. Two interesting applications of the newly obtained extension of classical Saalschutz's summation theorem are given.
GENERALIZATION OF KUMMER’S SECOND THEOREM with APPLICATIONS

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Abstract
The aim of this research paper is to obtain single series expression of
\[ e^{-\frac{x}{2}} \text{F}_1(\alpha; 2 \alpha + i; x) \]
for \( i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \), where \( \text{F}_1(.) \) is the function of Kummer. For \( i=0 \), we have the well known Kummer second theorem. The results are derived with the help of the generalized Gauss second summation theorem obtained earlier by Lavoie et al. In addition to this, explicit expressions of
\[ 2\text{F}_1[-2n, \alpha; 2 \alpha + i; 2] \]
and \[ 2\text{F}_1[-2n-1, \alpha; 2 \alpha + i; 2] \]
each for \( i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \) are also given. For \( i=0 \), we get two interesting and known results recorded in the literature. As an applications of our results, explicit expressions of
\[ e^{-x}\text{F}_1[\alpha; 2 \alpha + i; x] \times \text{F}_1[\alpha; 2 \alpha + j; x] \]
for \( i, j = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \) and
\[ (1 - x)^{-a}\text{F}_1(a, b; 2b + j; - \frac{2x}{1 - x}) \]
for \( j = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \) are given. For \( i=j=0 \) and \( j=0 \), we respectively get the well known Preece identity and a well known quadratic transformation formula due to Kummer. The results derived in this paper are simple, interesting, easily established and may be useful in the applicable sciences.

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On Some New Contiguous Functions Relations with Applications

MEDHAT A. RAKHA, ARJUN K. RATHIE, PURNIMA CHOPRA

Abstract

Contiguous relations for hypergeometric series contain an enormous amount of hidden information. Applications of contiguous relations range from the evaluation of hypergeometric series to the derivation of summation and transformation formulas for such series. In this paper, a new set of contiguous function relations are established. Applications of such relations to summation formulas and the theory of Jacobi polynomials are presented.
GENERALIZATIONS OF CLASSICAL SUMMATION THEOREMS FOR THE SERIES 2F1 AND 3F2 WITH APPLICATIONS

MEDHAT A. RAKHA, ARJUN K. RATHIE

Abstract

It is well known that classical summation theorems such as of Gauss, Gauss second, Kummer and Bailey for the series 2F1 and Watson, Dixon and Whipple for the series 3F2 play an important role in the theory of hypergeometric and generalized hypergeometric series. Applications of these summation theorems are well known.

The aim of this research paper is to provide the generalizations of the above mentioned classical summation theorems in the most general case.